MOS translinear circuits

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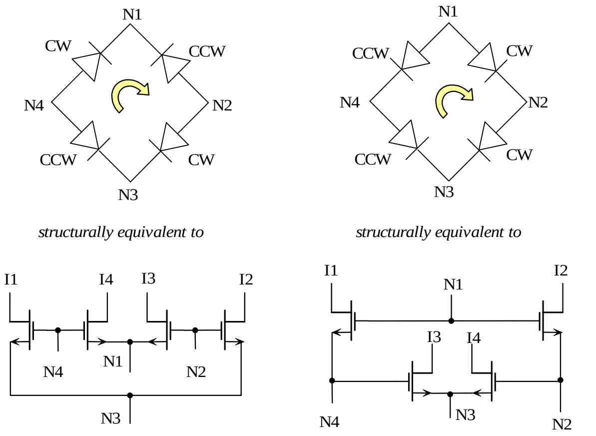
The translinear principle



Type 'A' Type 'B'

$$\prod_{CW} I = \prod_{CCW} I$$

The translinear principle

Translinear element $G = KI(V)$

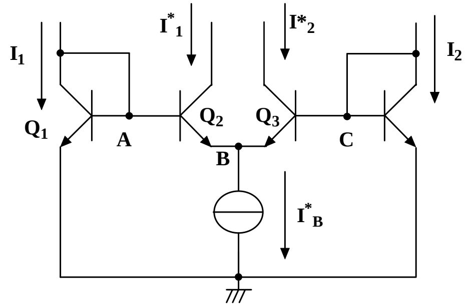
$$G = \frac{dI(V)}{dV} = KI(V) \Rightarrow I(V) = e^{KV}$$

Translinear principle

In a closed loop containing an even number of ideal junctions, arranged so that there are an equal number of clockwise-facing and counter-clockwise-facing polarities,with no further voltage generators inside this loop, the product of the current densities in the clockwise direction is equal to the product of the current densities in the counter-clockwise direction

$$\prod_{CW} J = \prod_{CCW} J$$

The translinear principle: example



$$I_C \cong A_E\left(e^{\frac{V_{BE}}{\phi_t}} - 1\right)J_{ES} \cong A_E J_{ES} e^{\frac{V_{BE}}{\phi_t}}$$

$$I_E \cong \frac{A_E J_{ES}}{\alpha_T} e^{\frac{V_{BE}}{\phi_t}}$$

A_E = emitter area,
J_{ES} = reverse saturation current density

Loop: ABC GND A (same geometry)

$$\boxed{I_1 I_2^* = I_1^* I_2 \qquad \frac{I_1}{I_1^*} = \frac{I_2}{I_2^*}}$$

If we set:

$$\Delta I = I_1 - I_2$$
$$I_B = I_1 + I_2$$
$$\Delta I^* = I_1^* - I_2^*$$
$$I_B^* = I_1^* + I_2^*$$

we can derive:

$$\boxed{\Delta I^* = \left(\frac{I_B^*}{I_B}\right) \cdot \Delta I}$$

MOS translinear circuits

$$I_D = \left(\frac{W}{L}\right) I_M' \exp\left(\frac{V_{GS}-V_M}{n\phi_t}\right)\left[1-\exp\left(\frac{V_{DS}}{\phi_t}\right)\right]$$

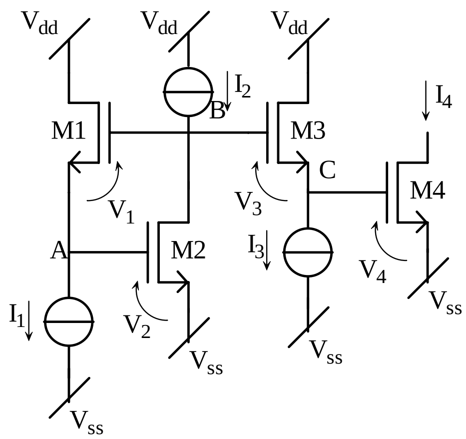
MOS drain current in weak inversion

$$I_D \cong \left(\frac{W}{L}\right) I_M' \exp\left(\frac{V_{GS}-V_M}{n\phi_t}\right) = \frac{W}{L} I_M' e^{-\frac{V_M}{\phi_t}} e^{\frac{V_{GS}}{n\phi_t}} = I_S e^{\frac{V_{GS}}{n\phi_t}} \qquad \text{where} \quad I_S = \frac{W}{L} I_M' e^{-\frac{V_n}{\phi_t}}$$

A MOS transistor in weak inversion is a translinear element

$$g_m = \frac{I_D}{n\phi_t}$$

MOS translinear circuits



loop Vdd-A-B-C-Vss

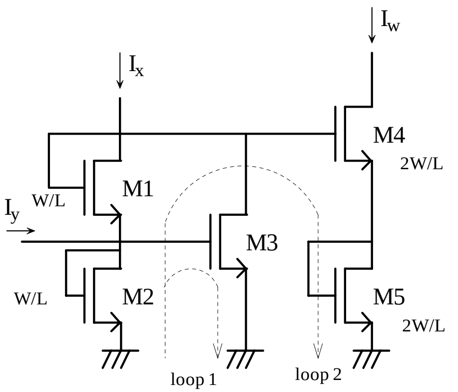
from KVL: $V_1+V_2-V_3-V_4=0$

$$\ln\left(\frac{I_1}{I_S}\right) + \ln\left(\frac{I_2}{I_S}\right) - \ln\left(\frac{I_3}{I_S}\right) - \ln\left(\frac{I_4}{I_S}\right) = 0$$

$$\ln\left(\frac{I_1 I_2}{I_S}\right) = \ln\left(\frac{I_4 I_3}{I_S}\right)$$

$$I_1 I_2 = I_3 I_4 \Rightarrow \boxed{I_4 = \frac{I_1 I_2}{I_3}}$$

MOS translinear circuits



loop 1: $I_{D2}=I_{D3}$ (current mirror)

loop 2: $I_{D1} I_{D2} = \frac{I_{D4}}{2}\frac{I_{D5}}{2}$

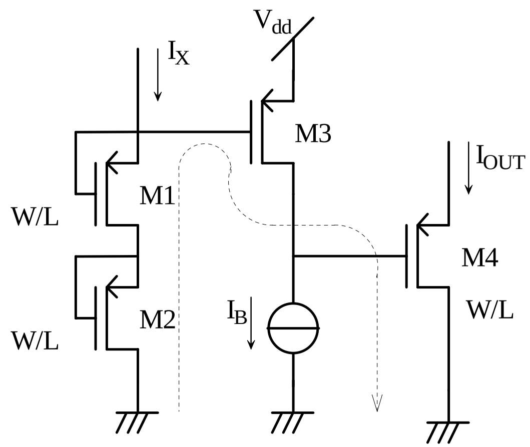
$$I_{D1} = I_y + I_{D2} = I_x - I_{D3} \qquad \textit{then:}\ \ 2I_{D2} = I_x + I_y$$

$$I_{D1} I_{D2} = \left\{-I_y + \frac{I_x+I_y}{2}\right\}\cdot\left\{\frac{I_x+I_y}{2}\right\} = \frac{I_x^2 - I_y^2}{2}$$

$$I_{D4} I_{D5} \left(= I_W^2\right) = 4 I_{D1} I_{D2}$$

then: $\boxed{I_W = \sqrt{I_x^2 - I_y^2}}$

MOS translinear circuits

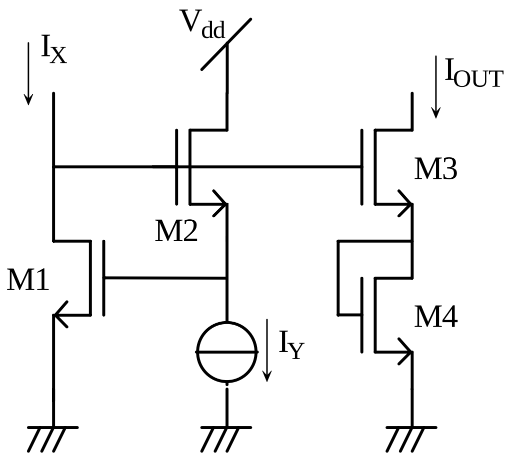


$$I_{D1} I_{D2} = I_{D3} I_{D4}$$

$$I_x^2 = I_B I_{OUT}$$

$$\boxed{I_{OUT} = \frac{I_x^2}{I_B}}$$

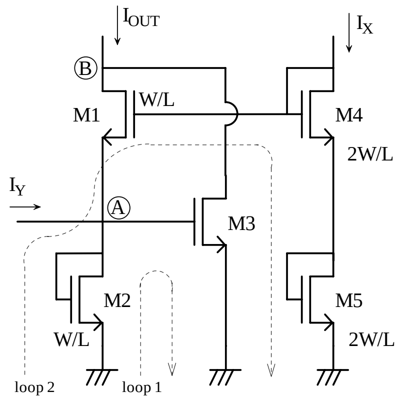
MOS translinear circuits



$$I_{D1}I_{D2} = I_{D3}I_{D4}$$

$$\boxed{I_{OUT} = \sqrt{I_x I_y}}$$

MOS translinear circuits



•loop1: $I_{D3} = I_{D2}$

$$\left.\begin{array}{l} \text{KCL node A}: \; I_{D2} - I_y - I_{D1} = 0 \\ \text{KCL node B}: \; I_{OUT} - I_{D3} - I_{D1} = 0 \end{array}\right\} \quad I_{D3} = I_{OUT} - I_{D2} + I_y$$

$$I_{D2} = I_{D3} \Rightarrow I_{D2} = \frac{I_{OUT} + I_y}{2} \;\; \text{e} \;\; I_{D1} = \frac{I_{OUT} - I_y}{2}$$

•loop2: $\frac{I_{D4}}{2}\frac{I_{D5}}{2} = I_{D1}I_{D2}$

$$\frac{I_x^2}{4} = \left(\frac{I_{OUT} + I_y}{2}\right)\left(\frac{I_{OUT} - I_y}{2}\right)$$

$$I_x = \sqrt{I_{OUT}^2 - I_y^2}$$

$$\boxed{I_{OUT} = \sqrt{I_x^2 + I_y^2}}$$

MOS translinear circuits

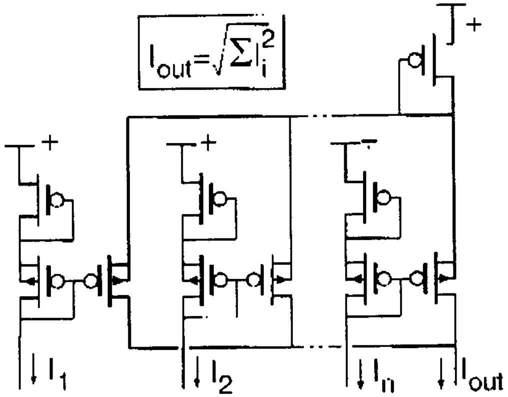
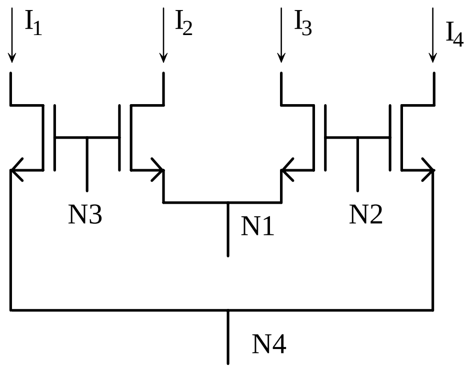


Fig. 2 Calculation of vector length by MOS translinear circuit.

MOS translinear circuits

•type A multiplier cell

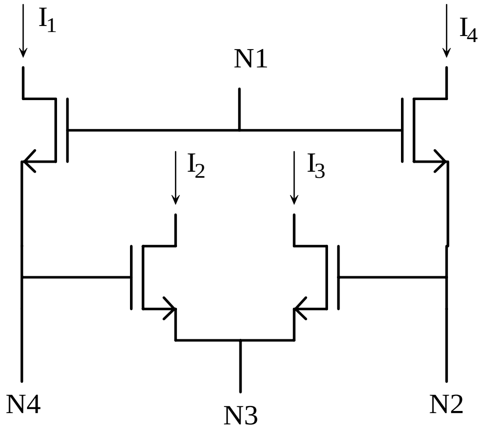


$$I_1 I_3 = I_4 I_2$$

$$\frac{I_1}{I_2} = \frac{I_4}{I_3}$$

MOS translinear circuits

•type B multiplier cell



$$I_1 I_3 = I_2 I_4$$

Device Mismatch

Device Mismatch

The device mismatch is due to the random variations of:

- V_{T0},
- "body effect" parameter γ
- $K = \frac{W}{L}\mu_n C_{OX}{}'$.

L > 2 μm

random variations have Gaussian distribution with zero mean value and variance which depends on W,L and D (distance between devices)

$$\sigma^2(\Delta V_{TO}) = \frac{A_{V_{TO}}{}^2}{WL} + S_{V_{TO}}{}^2 D^2$$

$$\sigma^2(\Delta\gamma) = \frac{A_\gamma{}^2}{WL} + S_\gamma{}^2 D^2$$

$$\left(\frac{\sigma(\Delta K)}{K}\right)^2 = \frac{A_K{}^2}{WL} + S_K{}^2 D^2$$

A_{VTO},A_γ,A_K,S_{VTO},S_γ,S_K technology parameters

Typical values:

TECHNOLOGY	TYPE	A_{VTO}	A_K	$(V_{GS}-V_T)_m$
		[mVμm]	[%μm]	[V]
2.5μm [Pel 89]	nMOS pMOS	30 35	2.3 3.2	2.6 2.2
1.2μm [Bas 95]	nMOS pMOS	21 25	1.8 4.2	2.3 1.2
0.7μm	nMOS pMOS	13 22	1.9 2.8	1.4 1.6

TECHNOLOGY	TYPE	S_{VTO}	S_K
		[μV/μm]	[ppm/μm]
2.5μm [Pel 89]	nMOS pMOS	4 4	2 2
1.2μm [Bas 95]	nMOS pMOS	0.3 0.6	3 5
0.7μm	nMOS pMOS	0.4 -	2 3

Device Mismatch

When taking account the absolute values :

$$\sigma(V_{T0}) = \sqrt{2}\,\sigma(\Delta V_{TO})$$

$$\sigma(\gamma) = \sqrt{2}\,\sigma(\Delta\gamma)$$

$$\left(\frac{\sigma(K)}{K}\right) = \sqrt{2}\left(\frac{\sigma(\Delta K)}{K}\right)$$

L < 1 μm

$$\sigma^2(\Delta V_{TO}) = \frac{A_{1VTO}}{WL} + \frac{A_{2VTO}}{WL^2} - \frac{A_{3VTO}}{W^2 L} + S_{VTO}{}^2 D^2$$

A_{1VTO},A_{2VTO} e A_{3VTO} technology parameters.

Typical values.

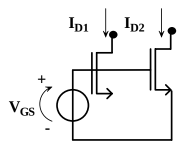
PARAMETER	nMOS	pMOS	
A_{1VT}	20	23	mV μm
A_{2VT}	19	20	mV $\mu m^{3/2}$
A_{3VT}	18	12	mV $\mu m^{3/2}$

Note: ΔV_{TO},$\Delta\gamma$,ΔK statistically indipendent.

Note: in the following V_{SB}=0

Device Mismatch

same Vgs



I_D= f (V_{DS},V_{GS},V_{T0},K)

$$\Delta I_D = \frac{\partial I_D}{\partial K}\Delta K + \frac{\partial I_{DS}}{\partial V_{TO}}\Delta V_{T0}$$

One can demonstrate that (both in weak and strong inversion):

$$\frac{\partial I_D}{\partial V_{T0}} = -\frac{\partial I_D}{\partial V_{GS}} = -g_m$$

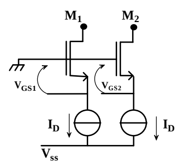
$$\frac{\partial I_D}{\partial K} = \frac{I_D}{K}$$

then:

$$\frac{\Delta I_D}{I_D} = \frac{\Delta K}{K} - \frac{g_m}{I_D} V_{T0}$$

$$\left(\frac{\sigma(\Delta I_D)}{I_D}\right)^2 = \left(\frac{\sigma(\Delta K)}{K}\right)^2 + \left(\frac{g_m}{I_D}\right)^2 \sigma^2(\Delta V_{VT0})$$

Same I_D:



output variable V_{GS}:

I_D-f(V_{GS},V_{DS},$V_{TO,}$K)=0

$$\frac{\partial I_D}{\partial K} - \frac{\partial f}{\partial K} = 0$$

$$\frac{\partial I_D}{\partial K} - \frac{\partial f}{\partial V_{GS}}\frac{\partial V_{GS}}{\partial K} = 0$$

$$\frac{\partial V_{GS}}{\partial K} = -\frac{\left(\frac{\partial I_D}{\partial K}\right)}{\left(\frac{\partial f}{\partial V_{GS}}\right)} = -\frac{I_D}{K g_m}$$

$$\frac{\partial V_{GS}}{\partial V_{TO}} = -\frac{\left(\frac{\partial I_D}{\partial V_{TO}}\right)}{\left(\frac{\partial I_D}{\partial V_{GS}}\right)} = 1$$

Device Mismatch

then (both in weak and in strong inversion):

$$\Delta V_{GS} = \Delta V_{T0} - \left(\frac{I_D}{g_m}\right)\left(\frac{\Delta K}{K}\right)$$

$$\sigma^2(\Delta V_{GS}) = \sigma^2(\Delta V_{T0}) + \left(\frac{I_D}{g_m}\right)^2\left(\frac{\sigma(\Delta K)}{k}\right)^2$$

<u>Strong inversion</u>

$$\frac{g_m}{I_D} = \frac{2}{V_{GS} - V_T} \quad \textit{then:}$$

$$\bullet\left(\frac{\sigma(\Delta I_D)}{I_D}\right)^2 = \left(\frac{\sigma(\Delta K)}{K}\right)^2 + \frac{4\sigma^2 V_{TO}}{(V_{GS} - V_T)^2} =$$

$$= \left(\frac{\sigma(\Delta K)}{K}\right)^2 + \frac{4\sigma^2 V_{TO}}{GVO^2}$$

to increase precision one has to increase the GVO

$$\bullet\,\sigma^2(\Delta V_{GS}) = \sigma^2(\Delta V_{TO}) + \frac{(V_{GS} - V_T)^2}{4}\left(\frac{\sigma(K)}{K}\right)^2$$

$D \cong 0$ *(i.e. close devices):*

$$\bullet\left(\frac{\sigma(\Delta I_D)}{I_D}\right)^2 = \frac{1}{WL}\left(A_K^2 + \frac{4A_{V_{TO}}^2}{GVO^2}\right) \cong \frac{1}{WL}\frac{4A_{V_{TO}}^2}{GVO^2}$$

$$\bullet\,\sigma^2(\Delta V_{GS}) = \frac{1}{WL}\left(A_{V_{TO}}^2 + \frac{GVO^2 A_K^2}{4}\right) \cong \frac{1}{W} A_{V_{TO}}^2$$

<u>Weak inversion</u>

$$\frac{g_m}{I_D} = \frac{1}{nV_T} \quad \textit{then:}$$

$$\bullet\left(\frac{\sigma(\Delta I_D)}{I_D}\right)^2 = \left(\frac{\sigma(\Delta K)}{K}\right)^2 + \frac{\sigma^2 V_{T0}}{(nV_T)^2}$$

$$\bullet\,\sigma^2(\Delta V_{GS}) = \sigma^2(\Delta V_{T0}) + (nV_T)^2\left(\frac{\sigma(K)}{K}\right)^2$$

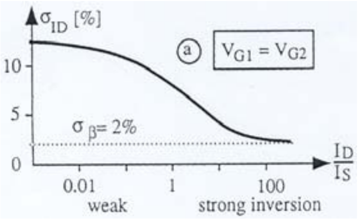
$D \cong 0$:

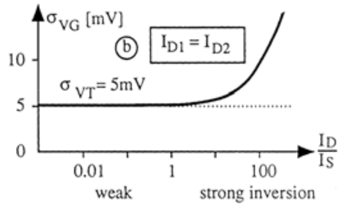
$$\bullet\left(\frac{\sigma(\Delta I_D)}{I_D}\right)^2 = \frac{1}{WL}\left(A_K^2 + \frac{A_{V_{T0}}^2}{(nV_T)^2}\right) \cong \frac{1}{WL}\frac{A_{V_{T0}}^2}{(nV_T)^2}$$

$$\bullet\,\sigma^2(\Delta V_{GS}) = \frac{1}{WL}\left(A_{V_{T0}}^2 + (nV_T)^2 A_K^2\right) \cong \frac{A_{V_{T0}}^2}{WL}$$

<u>Note</u>: the V_{T0} mismatch term is dominant on the K one

Device Mismatch





<u>Note</u>: β corresponds K and I_S to I_Z

<u>Note</u>: in general $|\Delta K_n| < |\Delta K_p|$ nMOS matching is better than pMOS one.