

Sensor-Based Measure Systems

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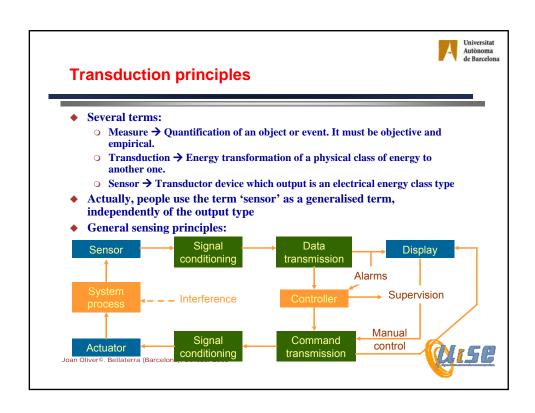


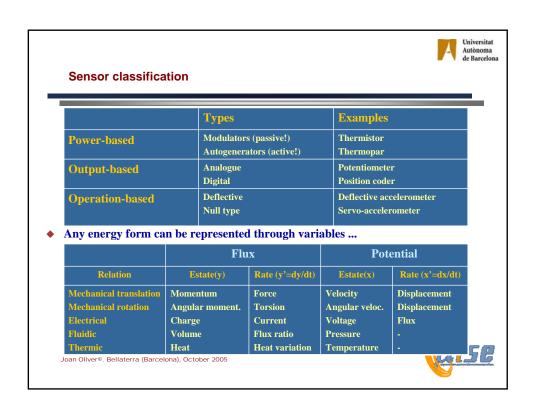
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Outline

- **♦** Transduction principles
- **♦** Resistive sensors
- **♦** Signal conditioning in resistive sensors
- **♦** Reactive sensors
- **♦** Signal conditioning in reactive sensors
- **♦** Digital sensors and smart sensors
- **♦** Interference and noise in amplifiers
- **♦** Special cases









Static properties of the measure systems

- **♦** Accuracy
- **♦** Precision: repeatibility + reproductibility
- Sensibility: $S(xa) = \frac{dy}{dx}\Big|_{x=xa}$
- ◆ Linearity/accuracy → hysteresis
- Systematic errors. The employed method affects the measure
 - Example: Measure of the potential decay in a resistance of tolerance 0.1%

Method 1: Using a voltmeter of 0.1% lecture accuracy → err = 0.1%

Method 2: Using an ammeter of 0.1% lecture accuracy:

$$\frac{dV}{V} = \frac{R \cdot dI + I \cdot dR}{V} = \frac{dI}{I} + \frac{dR}{R} \Longrightarrow \frac{\Delta V}{V} = \frac{\Delta I}{I} + \frac{\Delta R}{R} = \frac{0.1}{100} + \frac{0.1}{100} = 0.2\%$$

◆ Random errors → As the number of measures increases it tends to 0

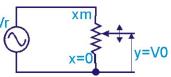
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Dynamic properties of the measure systems (I)

- Response of the systems in front of input signals
- Oynamic error → Difference between the real value and the measured value with static error to 0
- Usually measured with transient inputs: impulse, step, ramp, ...
- It is necessary to obtain the transfer function to study the dynamic properties.
- **♦** Zero order systems
 - Transfer function: $y(t) = k \cdot x(t)$
 - o k = static sensibility
 - o dynamic errors and null delay are null
 - o sensor without energy storage element



$$y = V_r \cdot \frac{x}{x_m} \Longrightarrow k = \frac{V_r}{X_m}$$





Dynamic properties of the measure systems (II)

First order systems

- - o k = 1/a0 = static sensibility
 - \circ τ = a1/a0 = time constant
 - $\begin{array}{l} \bigcirc \ \omega c = 1/\ \tau = \text{cut-off frequency (characterises the dynamic response)} \\ \bullet \ \text{Response to a step input(u(t))} \rightarrow \ k \cdot (1-e^{-t/\tau}) \end{array}$
 - with dynamic error: $e_d = y(t) k \cdot x(t) = 0; \quad delay = \tau$
 - with dynamic error:

Example: Thermometer based on a mass M with specific heat C (J/(Kg·K)), with transmission area = A and heat transfer coefficient h $(W/(m^2 \cdot K))$

$$h \cdot A \cdot (Te - Ti) \cdot dt - 0 = M \cdot c \cdot dTi$$

$$\frac{dTi}{dt} = \frac{h \cdot A}{M \cdot c} \cdot (Te - Ti)$$

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Dynamic properties of the measure systems (III)

Second order systems

• Transfer function:

$$a_{2} \frac{d^{2}y(t)}{dt^{2}} + a_{1} \frac{dy(t)}{dt} + a_{0} \cdot y(t) = x(t)$$

$$\frac{Y(s)}{X(s)} = \frac{k \cdot \omega_{n}^{2}}{s^{2} + 2 \cdot \xi \cdot \omega_{n} \cdot s + \omega_{n}^{2}}$$

$$\frac{Y(s)}{X(s)} = \frac{k \cdot \omega_n^2}{s^2 + 2 \cdot \xi \cdot \omega_n \cdot s + \omega_n^2}$$

static sensibility

$$k = \frac{1}{a_0}$$

o damping rate

$$\xi = \frac{a_1}{2 \cdot \sqrt{a_2 \cdot a_0}}$$

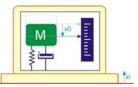
o natural frequency $w_n = 2 \cdot \pi \cdot f \implies \omega_n^2 = {a_0 \choose a_2}$

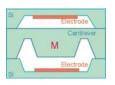
 $\xi = 1 \rightarrow \text{critical damping}$

 ξ < 1 \rightarrow subdamping

 $\xi > 1 \rightarrow \text{overdamping}$ Joan Oliver®. Bellaterra (Barcelona), October 2005

Example (accelerometer)





$$\mathbf{M} \cdot (\ddot{\mathbf{x}}_{i} - \ddot{\mathbf{x}}_{0}) = \mathbf{k} \cdot \mathbf{x}_{0} + \mathbf{B} \cdot \dot{\mathbf{x}}_{0}$$

Acceleration measure

$$\frac{X_0(s)}{\ddot{X}_i(s)} = \frac{X0(s)}{s^2 \cdot X_i(s)} = \frac{M}{K} \frac{K_M}{s^2 + s \cdot B_M + K_M} \rightarrow \begin{cases} k = M/K \\ \xi = B/2 \cdot \sqrt{K \cdot M} \\ \omega n = \sqrt{K_M} \end{cases}$$

Displacement measure

$$\frac{X_0(s)}{X_i(s)} = \frac{M}{K} \frac{\frac{K}{M} \cdot s^2}{s^2 + s \cdot B_{M} + \frac{K}{M}}$$





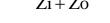
Input and output impedances

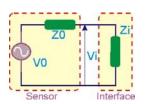
- Sensor output impedance determines the measuring input circuit impedance:
 - O An output in voltage demands a high input impedance in order to transport the sensor output voltage.

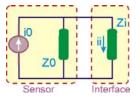
$$Vi = \frac{Zi}{Zi + Zo} \cdot Vo$$

O An output in current demands a low input impedance in order to transport the sensor output current.

$$Ii = \frac{Zo}{Zi + Zo} \cdot Io$$





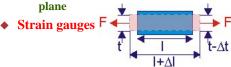




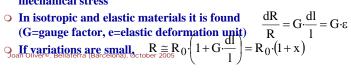
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Resistive sensors (I)

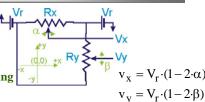
- Sensors based in the resistance variation. Several examples
- **Potentiometers**
 - O Example: Joystick: two potentiometers moving in four quadrants: situation of a point in a



- Piezoresistive materials: Resistance variation of a conductor or semiconductors due to $R = \rho \cdot \frac{l}{A} \Rightarrow \frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dl}{l} \frac{dA}{A}$ mechanical stress











Resistive sensors (II)

- **♦ RTD's (Resistive Temperature Detectors)**
 - Temperature detection by resistance variation
 - **O Example: Platinum temperature detectors (PTD)**

$$R = R_0 \cdot [1 + \alpha_1 \cdot \left(T - T_0\right) + \alpha_2 \cdot \left(T - T_0\right)^2 + \dots + \alpha_n \cdot \left(T - T_0\right)^n]$$

R0 → Reference temperature resistance

 α 1, α 2, α n \rightarrow Coefficient measured at reference temperatures (0°C, 100°C, ...)

O Example: Metals used as RTD in lineal range

$$R = R_0 \cdot [1 + \alpha_1 \cdot (T - T_0)], \ \alpha = \frac{R_{100} - R_0}{(100^{\circ}C)R_0}$$

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Resistive sensors (III)

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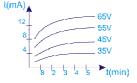
- **♦ Thermistor (Thermally Sensitive Resistor)**
 - Resistance temperature dependent semiconductors
 - They could be positive temperature coefficient (PTC) or negative temperature coefficient (NTC) thermistors
 - Several models

oTwo parameters (±0.3°C on 50°C range)
$$R_T = R_0 \cdot e^{\beta \cdot \left(\frac{1}{T} - \frac{1}{T_0}\right)}$$

$$R_T = R_0 \cdot e^{A + B_T + C_T / T^3}$$

 Input voltage (current) thermistors with constant heating dissipation are specially used in control circuits





<u> Uise</u>



Resistive sensors (IV)

♦ Magnetoresistive

- O Magnetoresistive effect: A magnetic field H applied to a conductor in conduction creates a Lorentz force $F = e \cdot v \times H$ over the e^{\cdot} , that alters the course of the e^{\cdot} . When the relaxation time is short, the e^{\cdot} displacement provokes a transversal electrical field (opposed to the e^{\cdot} movement Hall effect). The magnetoresistive effect appears at larger relaxation times, when the resistance increases.
- The magnetoresistive effect is lower than the Hall effect in the major part of the conductors. However, in anisotropic materials (f.e., in ferromagnetics), resistance is given by P. P. (P. P.) coc² 0
 - given by $R = R_{min} + (R_{max} R_{min})\cos^2\theta$ with Rmin appearing when the current is transversal to the magnetic field and Rmax when the current is parallel to the magnetic field.
- An external magnetic field provokes a rotation in the magnetisation axe so that

$$R = R_{\min} + \left(R_{\max} - R_{\min}\right) \left(1 - \left(\frac{H}{H_s}\right)^2\right)$$

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Resistive sensors (V)

♦ LDRs – Light Dependent Resistors

- The semiconductor resistance is a function of the received light
- O Radiation sensitive from 1mm to 10nm
- O Photonic energy is a function of the ratiation frequency

$$E = h \cdot f$$
, $\left(\lambda = \frac{c \cdot h}{E}\right)$, $h = 6.62 \cdot 10^{-34} J$ (Plank constant)

 $\begin{tabular}{ll} \hline O & Photoconductor resistance is a non linear relationship with the received illumination \\ \hline (Ev) & (A and α are photoresistor dependent parameters) \\ \hline \end{tabular}$

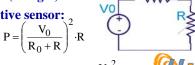
$$R = A \cdot E_v^{-\alpha}$$





Signal conditioning in resistive sensors (I)

- Environment conditions
 - Resistance as a parameter dependent function: $R = R0 \cdot f(x)$, f(0)=1
 - O Linear case: $R = R0 \cdot (1+x)$
 - Wide x range:
 - o from 10⁻¹² to 10⁻⁵ in strain gauges
 - o passing through 0 to -1 in linear potentiometers
 - to > 10000 in LDRs
 - Furthermore, sensors need power and are temperature dependent
- ♦ Measure of the resistance can be produced by 2-wire or 4-wire methods
- Measure can be deflective or null (bridges)
- **Equivalent Thévenin of the resistive sensor:** 2



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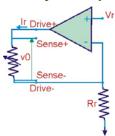
$$P \max(at R = R_0) = \frac{{V_0}^2}{4 \cdot R_0}$$



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Conditioning in resistive sensors (II)

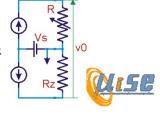
- Deflective simple method: to provide current and voltage measurement or to provide voltage and current measurement
- Example: NTC (negative temperature coefficient thermistor) sensor powered by constant current (voltage measurement)



$$v_o = I_r \cdot R = \frac{V_r}{R_r} \cdot R_o \cdot (1 + x)$$

 $Small \, signal \, analysis \, \Rightarrow \, v_s = v_o - I_r \cdot R_r \xrightarrow{\quad R_r = R_o \quad} = V_r \cdot \left(1 + x\right) - V_r = V_r \cdot x$

A second approach
$$v_0 = I \cdot (R - R_z) \xrightarrow{R_z = R_0} = I \cdot R_z \cdot x$$

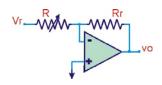


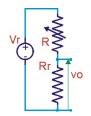


Voltage dividers (I)

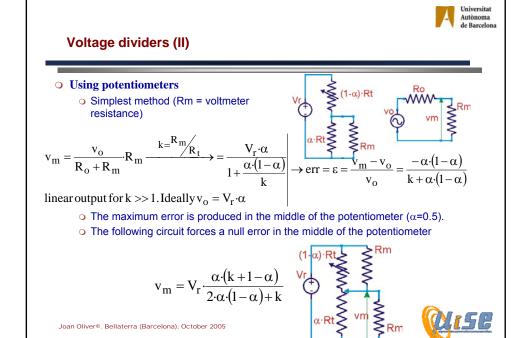
- It can be used in sensors with great resistance range and in non-linear sensors, where the vo-R non-linearity allows the linearization
- Vr and Rr must be chosen as a function of the desired operational range

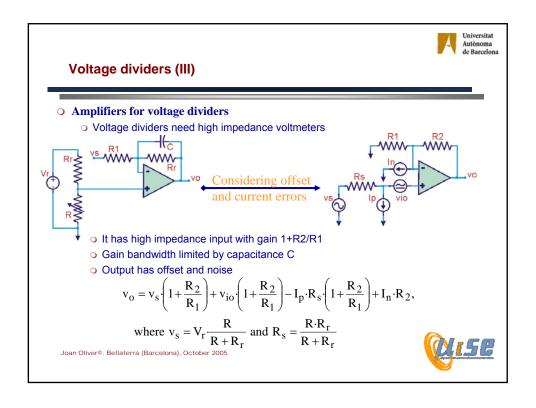
$$\begin{vmatrix} v_o = \frac{V_r}{R_r + R} \cdot R_r \\ R = \frac{V_o}{V_r - V_o} \cdot R_r \end{vmatrix} \rightarrow v_o = -Vr \cdot \frac{Rr}{R}$$

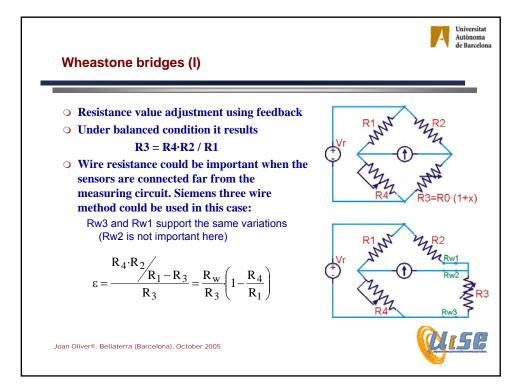


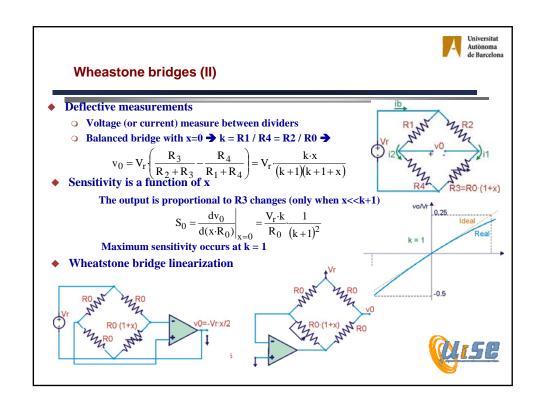


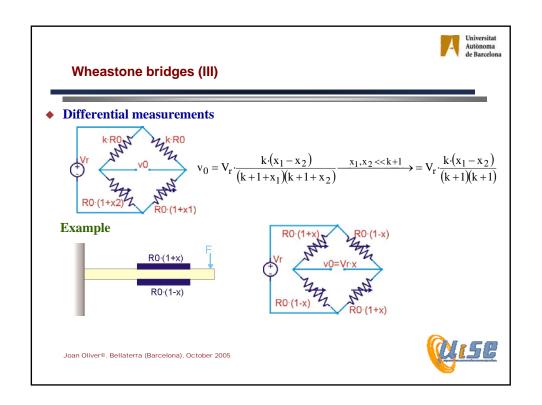


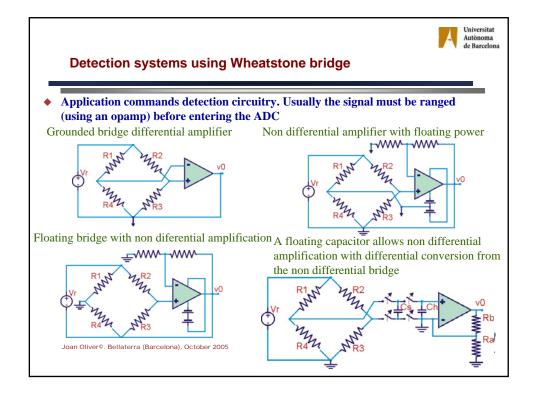


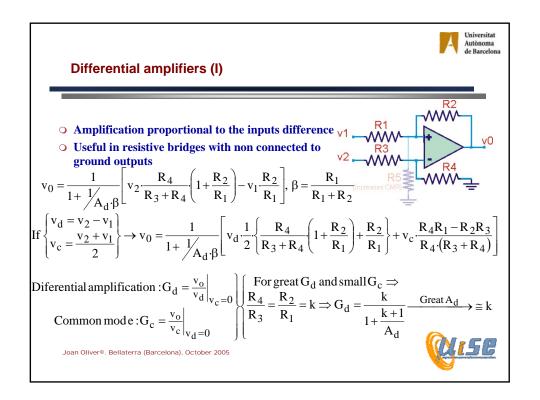














Differential amplifiers (II)

○ Gc = 0 is difficult to achieve: Common Mode Rejection Ratio

$$CMRR = \frac{G_d}{G_c} = \frac{1}{2} \cdot \frac{R_1 \cdot R_4 + R_2 \cdot R_3 + 2 \cdot R_2 \cdot R_4}{R_1 \cdot R_4 - R_2 \cdot R_3} \cong \frac{k+1}{4 \cdot t_r}$$

 t_r = resistace tolerance

• Input impedance must be considered

$$z_{i1} = \frac{v_1}{\frac{v_1 - v_n}{R_1}} = \frac{R_1}{1 - \frac{v_2}{v_1} \frac{R_4}{R_3 + R_4}}$$

$$z_{i2} = \frac{v_2}{\frac{v_2 - v_p}{R_3}} = R_3 + R_4$$

$$z_{i3} = \frac{v_2}{\frac{v_3 - v_p}{R_3}} = R_3 + R_4$$

$$z_{i2} = \frac{v_2}{\frac{v_3 - v_p}{R_3}} = R_3 + R_4$$

$$z_{i2} = \frac{(k+1)R_1}{2 \cdot k + 1}$$

$$z_{i2} = R_3 + R_4$$

$$z_{i2} = R_3 + R_4$$

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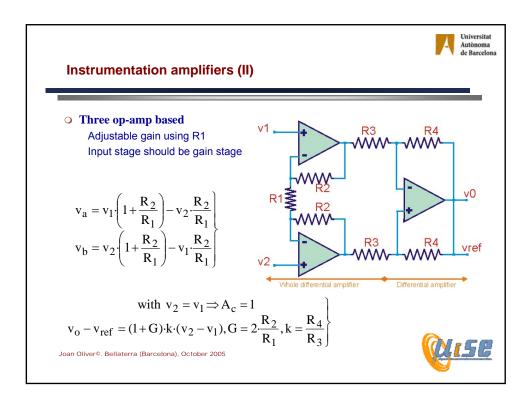
Two op-amp based

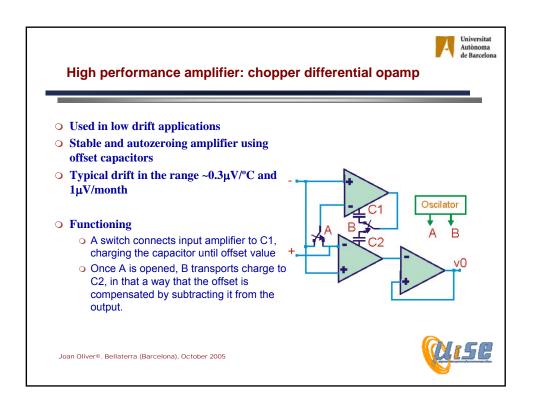
Differential amplifier with high gain and high input adjustable impedance (Rg) and low output impedance and offset

$$v_{o} = v_{d} \cdot \left(1 + k + \frac{R_{2} + R_{4}}{R_{g}}\right) + V_{ref}$$

$$v_{ref} = \frac{R_{4}}{V_{1}} + \frac{R_{3}}{V_{2}} + \frac{R_{3}}{V_{1}} + \frac{R_{2}}{V_{2}} + \frac{R_{3}}{V_{2}} + \frac{R_{3}}$$









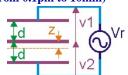
Reactant (capacitive) sensors

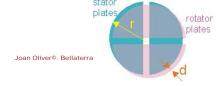
- Based on variable capacitance
 - O They can be linear or non-linear
 - O Used for measuring linear or rotational displacements, pressure, strain, ...

$$C\cong\epsilon_0\cdot\epsilon_r\cdot\frac{A}{d}(n-1),\ \epsilon_0=8.85pF/\,m,\ \epsilon_r\cong 1,\ n\ identical \ layers$$

- Based on differential capacitance
 - Used for displacement measures (ranging from 0.1pm to 10mm)

$$C1 = \frac{\varepsilon \cdot A}{d+z} \begin{cases} v_1 = V_r \cdot \frac{C2}{C1+C2} \\ C1 = \frac{\varepsilon \cdot A}{d-z} \end{cases} v_2 = V_r \cdot \frac{C1}{C1+C2} \begin{cases} v_1 - v_2 = V_r \cdot \frac{z}{d} \end{cases}$$





$$C1 = C3 = \frac{\varepsilon_0 \cdot \pi \cdot R^2}{4 \cdot d} \left(1 + \frac{2 \cdot \theta}{\pi} \right)$$

rotator
$$C1 = C3 = \frac{\epsilon_0 \cdot \pi \cdot R^2}{4 \cdot d} \cdot \left(1 + \frac{2 \cdot \theta}{\pi}\right)$$

$$C2 = C4 = \frac{\epsilon_0 \cdot \pi \cdot R^2}{4 \cdot d} \cdot \left(1 - \frac{2 \cdot \theta}{\pi}\right)$$



Reactant (inductive) sensors



- **Reluctant-inductive sensors**
- Reluctance → Magnetic flux quantity due to electric current

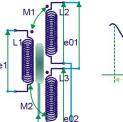
$$\begin{array}{c} \text{L} \rightarrow \text{Inductance, M} \rightarrow \text{Mutual inductance, R} \rightarrow \text{Magnetic reluctance} \\ L = n \cdot \frac{\varphi}{i} = n \cdot \frac{Fm/\Re}{i} = n \cdot \frac{n \cdot i/\Re}{i} = \frac{n^2}{\Re} \end{array}$$

For a spiral of I length and A section, (used as a metal detector) $\Re\cong$





LVDT: Linear Voltage Dependent Transformer





Mutual inductance variation between primary and secondary

Used in displacement, speed, acceleration, ...measurements

$$\mathbf{e}_{0} = \mathbf{i}_{2} \cdot \mathbf{R}_{L} = \frac{\mathbf{s} \cdot (\mathbf{M}_{1} - \mathbf{M}_{2}) \mathbf{e}_{1} \cdot \mathbf{R}_{L}}{2 \cdot \mathbf{s}^{2} \cdot \mathbf{L}_{1} \mathbf{L}_{2} + \mathbf{s} \cdot (\mathbf{R}_{2} \cdot \mathbf{L}_{1} + 2 \cdot \mathbf{R}_{1} \cdot \mathbf{L}_{2}) + \mathbf{R}_{1} \cdot \mathbf{R}_{2}}$$



Other sensors

- Magnetic sensors
 - Ac tachometers
 - Hall effect sensors
 - Synchros, resolvers, inductosyn
 - Magnetoelastic and magnetoresistive
 - Pulse-wire sensors and wiegand sensors
 - Flux-gate
 - **○** SQUID → Superconducting Quantum Interference Device's
- **♦** Thermopar autogenerator sensors
- Pyroelectric autogenerator sensors
- **♦** Photovoltaic sensors

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Conditioning in capacitive sensors (I)

- **♦** Sensor reactance measurement
 - Usually implies the measure of low capacitances with great stray capacitances
 - Two main circuits for
 - Sensors with linear admitance change

For example: layer displacement capacitance (in a capacitor) sensor

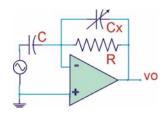
Constant current is applied to the sensor

R polarises the sensor, and it must be much greater than sensor impedance

$$C_x = \varepsilon \cdot \frac{A}{d \cdot (1+x)} = \frac{C_o}{1+x}$$

Considering that R does not influence the measure

$$v_o = -v_e \cdot \frac{Z_x}{Z} = -ve \cdot \frac{C}{C_o} \cdot (1+x)$$





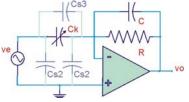


Conditioning in capacitive sensors (II)

Sensors with linear impedance change

Constant voltage is applied to the sensor Usually charge amplifier is used.

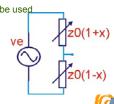
$$v_o = -v_e \cdot \frac{C_x}{C}$$



In the case of differential sensors voltage dividers can be used
The constant term dominates when x<<1
Interference errors in both sensors are canceled

$$v_{o} = v_{e} \cdot \frac{z_{o} \cdot (1+x)}{z_{o} \cdot (1-x) + z_{o} \cdot (1+x)} = v_{e} \cdot \frac{1+x}{2}$$

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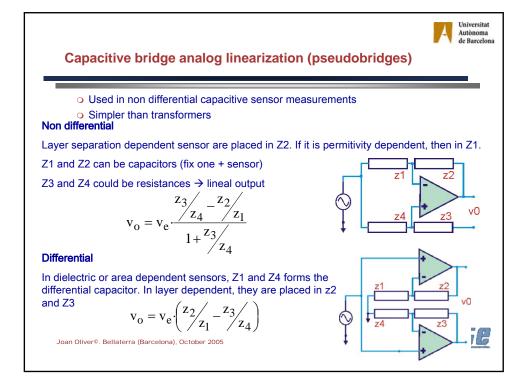


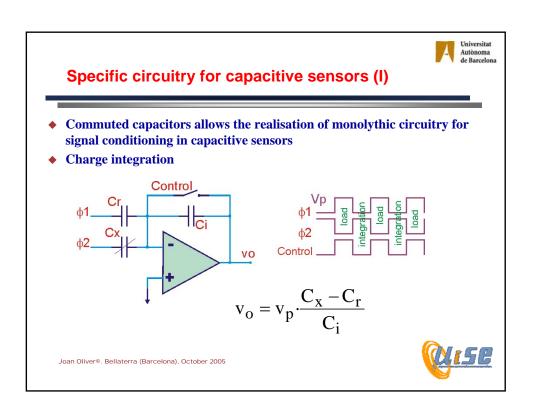


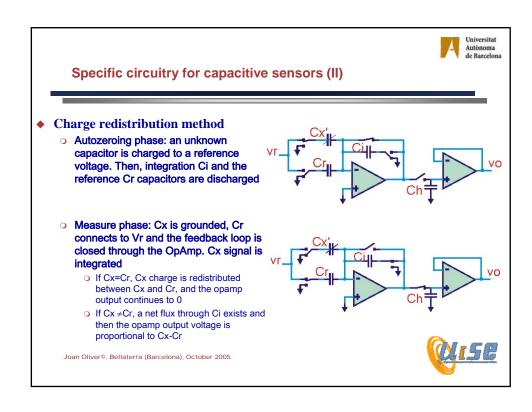
ac bridges

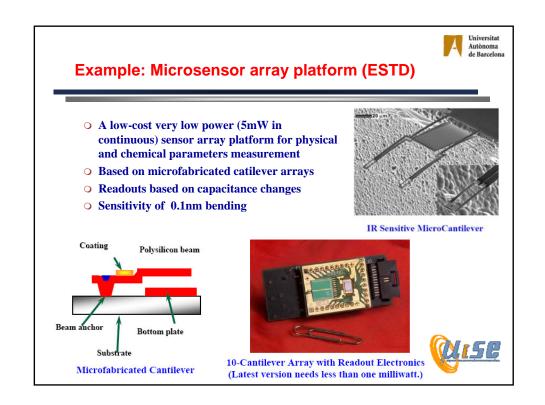
- As in voltage dividers,
 - ullet Bridges with unique sensor ullet Non linear output
 - \bullet Differential bridges \Rightarrow linear output and cancellation of coincident errors in both sensors
- Bridges in inductive sensors
 - Similar as in resistance bridge sensors
- Bridges in capacitive sensors
 - It is recommended do not use resistances because the parasitic impedances in capacitors can be as large as those and, thus, provoking important errors.
 - o use of Blumblein bridges or transformers
 - Cs1 and Cs2 with small influence
 - N2 and N3 determine the voltage rate between z1 and Z2
 - 0.1pF over 50pF values with 1nF stray capacitances can be detected
 - With values z1=z0·(1-x) and z2=z0·(1+x), and big detector input impedance, vd=vs=v2·x/2

Wise.





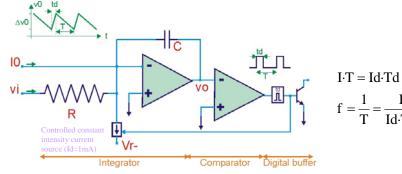






Towards digital processing (I)

♦ Voltage-to-frequency conversion (VFC's)



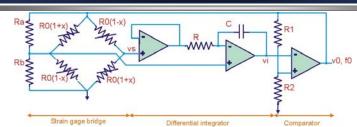
- Highly linear, great resolution and good noise immunity. Accuracy dependent on the component errors
- O A bit slow conversion

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Quantity-to-frequency direct conversion (I)



Comparator at low level $\rightarrow \overline{V_0} \Rightarrow v_s = \overline{V_0} \cdot x$

$$t = t0 \text{ (and } t_{\uparrow}) : v_i(t) = \frac{V_0 \cdot x}{R \cdot C} \cdot t + V_i(0)$$

Comparator at high level $\rightarrow V_0 \Rightarrow v_s = V_0 \cdot x$

$$t = T_1 \left(comparator \, commutes \right) : v_i(T_1) = \frac{V_0 \cdot x}{R \cdot C} \cdot T_1 + V_i(0) = V_0 \cdot \frac{R2}{R1 + R2}$$

$$From \, T_1 : v_i(t) = \frac{\overline{V_0} \cdot x}{R \cdot C} \cdot \left(t - T_1\right) + V_0 \cdot \frac{R2}{R1 + R2}$$

$$t = T_2: v_i(T_2) = \overline{\frac{V_0}{R \cdot C}} \cdot \left(T_2 - T_1\right) + V_0 \cdot \frac{R2}{R1 + R2} = \overline{V_0} \cdot \frac{R2}{R1 + R2} \Longrightarrow V_i(0) = \overline{V_0} \cdot \frac{R2}{R1 + R2}$$



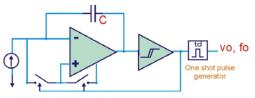


Quantity-to-frequency direct conversion (II)

Thus:
$$T_1 = (V_0 - \overline{V_0}) \frac{R_2}{R_1 + R_2} \frac{R \cdot C}{V_0 \cdot x}$$

$$T_2 = (V_0 - \overline{V_0}) \frac{R_2}{R_1 + R_2} \frac{R \cdot C}{x} \left(\frac{1}{V_0} - \frac{1}{\overline{V_0}} \right)$$
So: $f0 = \frac{x}{R \cdot C} \frac{R_1 + R_2}{R_2} \frac{-V_0 \cdot \overline{V_0}}{(V_0 - \overline{V_0})^2} \xrightarrow{\text{if } V_0 = -\overline{V_0}} f0 = \frac{x}{R \cdot C} \frac{R_1 + R_2}{4 \cdot R_2}$
Thus, the solution is voltage independent

- independent.
- Current output sensors have easy frequency conversion using direct current integration. For example, TSL220 (TI) integrated circuit performs the conversion light-tocurrent. The output integration time is proportional to photodiode **current.** pan Oliver©. Bellaterra (Barcelona), October 2005

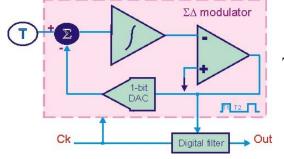




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Quantity-to-frequency direct conversion (III)

- The TMP03(°C)/TMP04(°F) integrated circuits (Analog Devices) have a $\Sigma\text{--}\Delta$ modulator to perform the temperature conversion to digital output
- Accuracy is a function of the clock frequency and counter resolution.

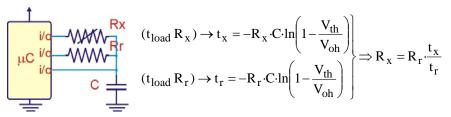


 $T(^{\circ}C) = 235 - \frac{400 \cdot T_1}{T_2}$

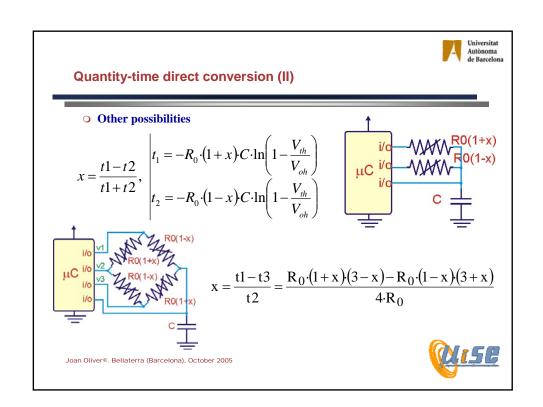


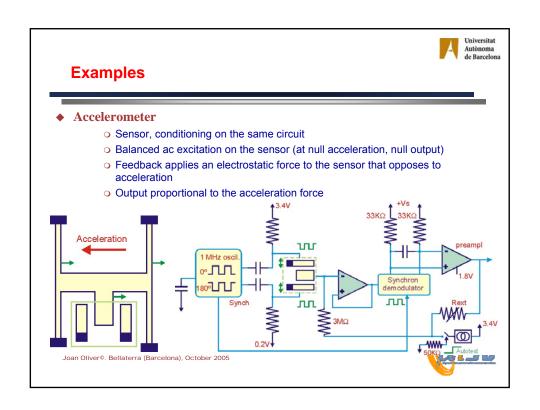
Quantity-time direct conversion (I)

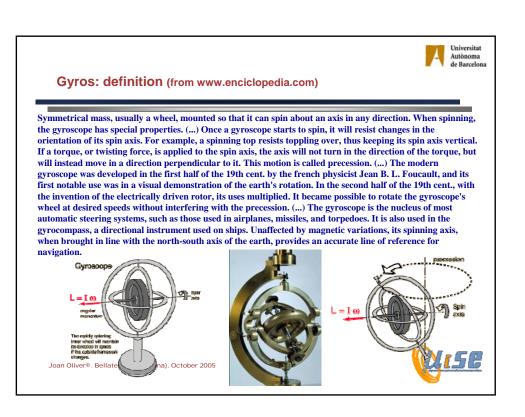
- The comparison of an unknown resistance to a known one is a simple measuring technique. This can be done using the load/unload capacitance time, and it can be done using a uC.
- Care must be taken with the I/O port finite resistance
- C determines the resolution and must be chosen depending on the expected counting.













Gyros: math equations (I)

(From Engineering Mechanics - Statics and Dynamics, R. C. Hibbeler)

- Newton's Law states that the sum of the *external* forces acting on a particle equals the particle's mass times its acceleration. Actually, Newton's original formulation related the external forces to the particle's *linear momentum*. (1)
- If we chose a reference point O and r is a position vector to the particle, we can take the cross product
 of both sides of this equation to get an expression that relates the moment of the forces (Mo) acting on
 the particle to the angular momentum (Ho) of the particle with respect to the reference point O. (2)
- 3. After several equalities, it is found that, given a moving particle, the Sum of the Moments about a point O is equal to the time rate of change of the particle's angular momentum. (3)

$$\sum \vec{F} = m \cdot \vec{a} = m \cdot \vec{v}' \tag{1}$$

$$\sum \vec{M}_{o} = \vec{r} \times \sum \vec{F} = \vec{r} \times m \cdot \vec{v}' \quad (2)$$

$$\sum \vec{\mathbf{M}}_{0} = \vec{\mathbf{H}}'_{0} \tag{3}$$

This equation (derived for a particle) is also valid for a system of particles. That is, the Sum of the Moments about point O due to the external forces acting on a system of particles is equal to the time rate of change of the angular momentum of the system of particles about this same reference point O.

So, this equation applies to the analysis of a gyroscope.

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Gyros: math equations (II)

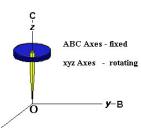
- 4. Now, we need an expression for the angular momentum Ho or its time derivative H'o with attributes that can be physically measured such as mass, radius, angular velocity, and angular acceleration. Equation (4) gives ΔHo for a particle in the body having an incremental mass Δm and an angular velocity ω.
- 5. Summing for all the particles of the body, we obtain equation (5)
- 6. And, solving it in the case of a principal axes of inertia (a particular case where the coordinate system has at least two of the three orthogonal planes defined by the coordinate system as axes of symmetry of the body), we obtain equation (6).

$$\left[\Delta \vec{H}_{o}\right]_{j} = \vec{r} \times \Delta m_{j} \cdot \vec{v}_{j} = \vec{r} \times (\vec{w} \times \vec{r})_{j} \cdot \Delta m_{j} \tag{4}$$

$$\vec{H}_{o} = \int_{\vec{H}_{o}} d\vec{H}_{o} = \sum \left[\Delta \vec{H}_{o} \right]_{j} = \int_{m} \vec{r} \times (\vec{w} \times \vec{r}) dm \qquad (5)$$

$$\vec{\mathbf{H}} = \vec{\mathbf{I}} \cdot \vec{\mathbf{w}}, \quad \vec{\mathbf{I}} = \left(\mathbf{I}_{XX}, \mathbf{I}_{VV}, \mathbf{I}_{ZZ}\right) \tag{6}$$

Angular momentum **Ho** can now be calculated since the angular velocity can be measured and the *moments of inertia* (I), which depend only upon the mass and physical dimensions of the body, can be looked up in a table or calculated by hand.



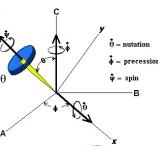


Gyros: math equations (III)

- 7. Analytical gyroscope results can be obtained choosing a coordinate system such that at least two of the three orthogonal planes defined by the coordinate system are axes of symmetry for the gyroscope. Then all the products of inertia will become zero, and we only have to include moments of inertia in our calculations. To accomplish this, we will use a *rotating coordinate system* with origin at the pivot point of the gyro. The rotating coordinate system will follow the gyro's nutation and precession but not its spin. We will call the angular velocity of the rotating reference Ω = nutation + precession = θ ' + ϕ ' and label the rotating axes as the xyz axes. We will also have a fixed reference coordinate system which will also have its origin at the pivot point of the gyro and we will call this the ABC axes. The angular velocity of the gyro with respect to the fixed ABC axes will be ω = nutation + precession + spin = θ ' + ϕ ' + ψ '. The angles θ , ϕ and ψ are called *Euler angles*
- 8. So, $H_o \rightarrow H_o' \rightarrow \Sigma M$ can be calculated: $\sum M_x = I_x \cdot \theta'' I_y \cdot (\phi')^2 \cdot \cos \theta \cdot \sin \theta + I_z \cdot \phi' \cdot \sin \theta \cdot (\phi' \cdot \cos \theta + \psi')$ $\sum M_y = I_y \cdot (\phi' \cdot \theta' \cdot \cos \theta + \phi'' \cdot \sin \theta) I_z \cdot \theta' \cdot (\phi' \cdot \cos \theta + \psi') + I_x \cdot \phi' \cdot \theta' \cdot \cos \theta$ $\sum M_z = I_z \cdot (-\phi' \cdot \theta' \cdot \sin \theta + \phi'' \cdot \cos \theta + \psi'') I_x \cdot \phi' \cdot \theta' \cdot \sin \theta + I_y \cdot \theta' \cdot \phi' \cdot \sin \theta$

In general, it would be very hard to arrive at a solution that satisfies these equations, however ...

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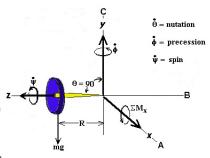


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Gyros: math equations (IV)

7. ...in the specific case where the precession ϕ' is constant, the spin ψ' is constant, and the nutation angle $\theta = 90^{\circ}$ is a constant 90 degrees as shown in figure below, the solution becomes quite easy:

8. So the only moment in this case is the moment about the x-axis. There are no negative signs in this equation and we consistently use the *right-hand-rule* in expressing all vector quantities, therefore, all of the vectors - the sum of the moments about the x-axis (ΣMx), the precession of the gyro about the y-axis (φ'), and the spin of the flywheel about the z-axis (ψ') all act along their respective positive axes as indicated in the figure.





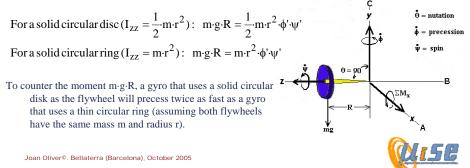


Gyros: math equations (V)

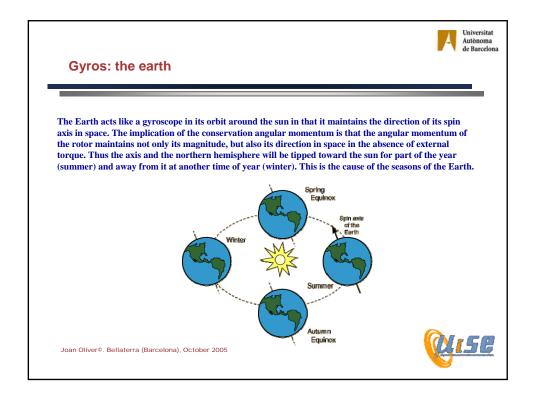
 As the only moment about the x-axis is the moment resulting from the weight of the flywheel, and if the flywheel is at a distance R from the pivot point (origin O), and recalling that Iz = Izz, we have:

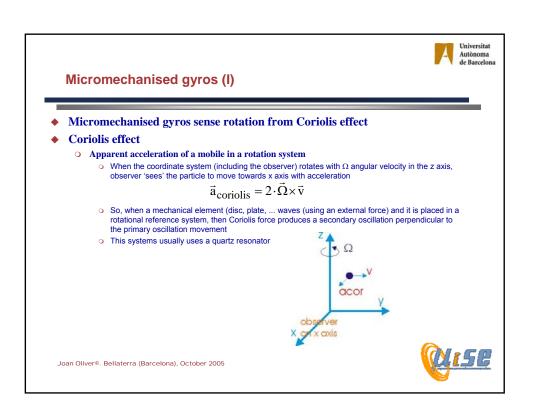
$$m \cdot g \cdot R = I_{zz} \cdot \phi' \cdot \psi'$$

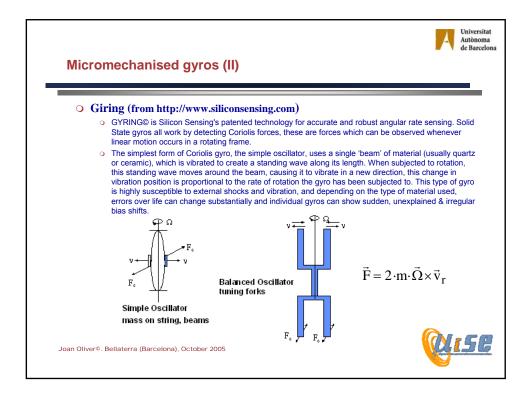
10. This equation tells us that if the flywheel is lying at 90° at a distance R from the pivot point and the flywheel is spinning at a constant angular velocity y', then the gyro will not topple over as might be expected, but instead, it will precess about the positive y-axis at a constant angular velocity of φ'.

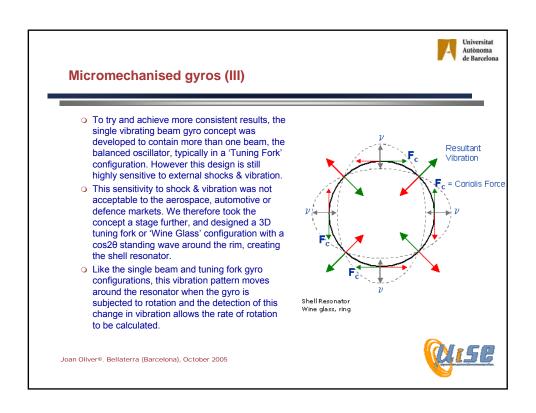


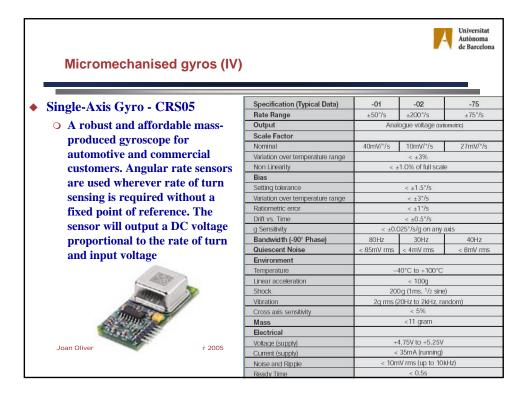
Universitat Gyros: precession spinning top A rapidly spinning top will preces in a direction determined by the torque exerted by its weight. The precession angular velocity is inversely proportional to the spin angular velocity, so that the precession is faster and more pronounced as the top slows down This process involves a considerable number of physical and mathematical concepts. The angular momentum of the spinning top is given by its moment of inertia times its spin speed but this exercise requires an understanding of it's vector nature. A torque is exerted about an axis through the top's supporting point by the weight of the top acting on its center of mass with a lever arm with respect to that support point. Since torque is equal to the rate of change of angular momentum, this gives a way to relate the torque to the precession process. From the definition of the Lsind angle of precession, the rate of change of the precession angle q can be expressed in terms of the rate of change of angular momentum and hence in terms of the torque. The expression for precession angular velocity is valid only under the conditions where the spin angular velocity w is much greater than the precession angular velocity wP. When the top slows down, the top begins to wobble, an indication that more complicated types of motion are coming into play. Joan Oliver©. Bellaterra (Barcelona), October 2005













Gyros (... pure technology)

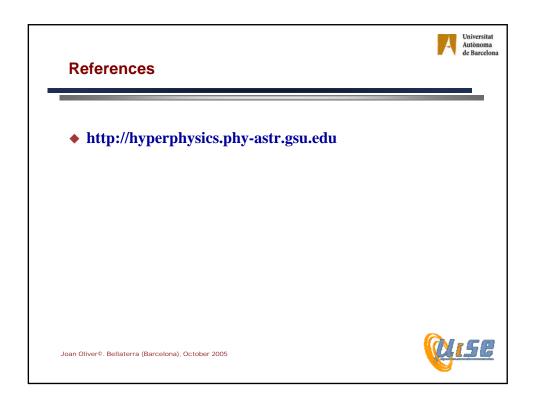
o iBOT™

- The Johnson & Johnson iBOT™ Mobility System provides new levels of personal freedom and accessibility for people with disabilities, it is a unique gyro-balanced mobility device that can operate on either four or two wheels, stabilising the user by automatically adjusting and balancing itself.
- Allows you to climb up and down stairs with or without assistance, allowing you to gain access to previously difficult places to reach.
- It can climb curbs as high as 4 inches and travel over grass, gravel, sand and other forms of uneven terrain.







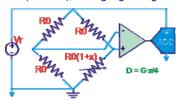






Power supply in Wheastone bridges

- O DC power in Wheatstone bridges must be taken into account: $\frac{dv_0}{v_0} = \frac{dV_r}{V_r}$
- O Precision measurements require minimisation of power supply errors
- Several options:
 - Making an output rate proportional between output and power supply
 - Using opamp with high PSRR
 - Power supply regulators in the bridge power supply could afford good solutions in applications with high input power supply (10V, for example) and low measurement urrents (< 20mA), as in gauge bridges.



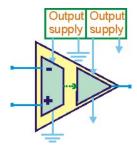
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Isolation amplifier

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- Used in highly noisy environments
- Optocoupler based amplifier







Electromagnetic sensors

♦ Based on Faraday law $E = -n \cdot \frac{d\phi}{dt}$

 Example: ac tachometer: a n spiral circuit that generates a voltage when subject a magnetic field, with output voltage and frequency variable.

$$\begin{split} E = -n \cdot \frac{d\varphi}{dt} &= -n \cdot \frac{d \big(B.A.\cos\theta\big)}{dt} = n \cdot B \cdot A \cdot \sin\theta \cdot \frac{d\theta}{dt} \bigg] \\ & \rightarrow e = n \cdot B \cdot A \cdot \omega \cdot \sin\int\omega \cdot d\omega \\ & \omega = 2 \cdot \pi \cdot n = \frac{d\theta}{dt} \\ & = n \cdot B \cdot A \cdot 2 \cdot \pi \cdot n \cdot \sin\big(2 \cdot \pi \cdot n \cdot t\big) \end{split}$$

♦ Hall effect sensors

 Differential of potential generation in a semiconductor into a magnetic field and with a current that crosses the semiconductor perpendicular to the magnetic field

$$\text{Coefficient Hall is given by } \\ A_H = \frac{V_H \cdot t}{I \cdot B} \\ \text{Used as a distance and speed detectors}$$



